

ENTRANCE EXAMINATIONS PRE-TOP UNIVERSITIES TYPE 1

120 minutes

READ THESE INSTRUCTIONS FIRST

1. Write your full name, class, school and phone numbers on all the work you hand in
2. Answer **all** the questions
3. The number of marks is given in brackets [] at the end of each question or part question
4. The total number of marks for this paper is 70

1. a. Simplify $\frac{2}{a} - \left(\frac{a+1}{a^3-1} - \frac{1}{a^2+a+1} - \frac{2}{1-a} \right) : \frac{a^3+a^2+2a}{a^3-1}$ [5]

b. Let $x = \sqrt{37 - 20\sqrt{3}}$. Find the value of $\frac{x^4 - 9x^3 + 5x^2 - 7x + 68}{x^2 - 10x + 19}$ [5]

2. a. Solve the inequality $\frac{x-10}{x+2} > x-5$ [5]

b. Hence, solve the inequality $\frac{\sqrt{x}-11}{\sqrt{x}+1} > \sqrt{x}-6$ [5]

3. An Arithmetic progression has 889 terms. The sum of all the even-numbered terms of the progression is 408840. The 1st term, 9th terms and the 21st term of the progression are three consecutive terms of a geometric progression. Find the first terms, and the common difference of the arithmetic progression [6]

4. Given that $\sqrt[3]{17 - \frac{27}{4}\sqrt{6}}$ and $\sqrt[3]{17 + \frac{27}{4}\sqrt{6}}$ are the roots of the equation $x^2 - ax + b = 0$

Find the value of a and b [5]

5. Find the values of x which satisfy the equation $(x^2 - 5x + 5)^{x+5} = 1$ [6]

6. Find the equation of quadratic function that value positive for $-7 < x < 1$ and the distance of vertex and origin is 5. [5]

7. Let $f(x) = \frac{10x+1}{10-100x}$. Set $f^n = f \circ f \circ \dots \circ f$ where composition f repeated n times

Find the value of $f\left(\frac{1}{2}\right) + f^2\left(\frac{1}{2}\right) + f^3\left(\frac{1}{2}\right) + \dots + f^{6000}\left(\frac{1}{2}\right)$ [5]

8. Solve this system equation

$$\begin{cases} x + y + z = 13 \\ x^2 + y^2 + z^2 = 91 \\ y^2 = xz \end{cases} \quad [6]$$

9. Evaluate the sum

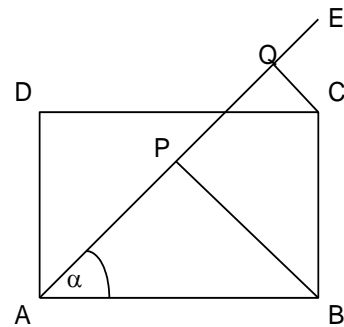
$$\frac{3!+4!}{2(1!+2!)} + \frac{4!+5!}{3(2!+3!)} + \dots + \frac{2018!+2019!}{2017(2016!+2017!)}$$

Where $n! = n \cdot (n-1) \cdot (n-2) \dots 3 \cdot 2 \cdot 1$

10. ABCD is a rectangle where $AB = x$ and $BC = y$.

BP and CQ are perpendicular to AE with angle BAE is α .

Show that : $AQ = x \cdot \cos \alpha + y \cdot \sin \alpha$



[4]

[6]

11. A $(-1, 4)$, B $(2, 7)$, D $(1, 0)$ and C are the four vertices of a parallelogram. The E lies on BC such that $BE = \frac{1}{3} BC$. Lines are drawn, parallel to the y-axis, from A meet the x-axis N and from E to meet CD at F.

- Calculate the coordinates of C and of E.
- Find the equation DC and calculate the coordinates of F.
- Explain why AEFN is a parallelogram and calculate its area !

[6]

