COMBINATORICS (KOMBINATORIK)

Wardaya College Winter Camp Olympiad 2017

1. A Farmer has a flock of n sheep, where $2000 \leq n \leq 2100$. The farmer puts some number of the sheep into one barn and the rest of the sheep into a second barn. The farmer realizes that if she were to select two different sheep at random from her flock, the probability that they are in different barns is exactly $\frac{1}{2}$. Determine the value of n

2. Ten lockers are in a row. The lockers are numbered in order with the positive integers 1 to 10. Each locker is to be painted either blue, red or green subject to the following rules:

   - Two lockers numbered m and n are painted different colours whenever $m - n$ is odd.
   - It is not required that all 3 colours be used.

   In how many ways can the collection of lockers be painted?

3. Find the coeffiecient of x^k , $k \geq 18$ in the expansion
   $\left(x^3 + x^4 + x^5 + ...\right)^6$

4. Let $X = \{1, 2, 3, ..., 100\}$ and $S = \{(a.b.c)\,|a, b, c, \in X, a < b \text{ and } a < c\}$
   Find $|S|$

5. Let T denotes the 15 element set $\{10a + b :\ a, b \in \mathbb{Z},\ 1 \leq a < b \leq 6\}$. Let S be a subset of T in which all six digits 1, 2, ..., 6 appera and in which no three element together use all these six digits. Determine the largest possible size of S

6. Albert and Betty are playing the following game. There are 100 blue balls in a red bowl and 1100 red balls in a blue bowl. In each turn a player must make on the following moves :

   (a) Take two red balls from the bowl and put them in the red bowl.
   (b) Take two blue balls from the red bowl and put them in the blue bowl.
   (c) Take two balls of different colors from one bowl and throw the balls away.

   They take alternate turns and Albert starts. The player who first take the last red ball from the blue bowl or the last blue ball from the red bowl wins. Determine who has a winning strategy.

7. In how many ways can we paint 16 seats in a row, each red or green, in such a way that the number of consecutive seats painted in the same colour is always odd?

8. For an upcoming international mathematics contest, the participating countries were asked to choose from nine combinatorics problems. Given how hard it usually is to agree, nobody was suprised that following happened:

   (a) Every country voted for exactly three problems.
   (b) Any two countries voted for different sets of problems.
   (c) Given any three countries, there was a problem none of them voted for.

   Find the maximal possible number of participating countries.

9. What is the least possible number of cells that can be marked on an $n \times n$ board such that for each $m > \frac{n}{2}$ both diagonals of any $m \times m$ sub-board contain a marked cell?

10. Let T denote 15-element set $\{10a + b :\ a,\ b \in \mathbb{Z},\ 1 \leq a < b \leq 6\}$. Let S be a subset of T in which all six digits 1, 2, ... , 6 appear and in which no three elements together use these six digits. Determine the largest possible size of S.