Matematika SMA - GEOMETRY (GEOMETRI)

Wardaya College Winter Camp Olympiad 2017

1. Let ABC be a triangle with circumcircle ω and ℓ a line without common points with ω. Denote by P the foot of the perpendicular from the center of ω and ℓ. The sideilines BC, CA, AB intersect ℓ at the points X, Y, Z different from P. Prove that the circumcircles of the triangles AXP, BYP and CZP have a common point different from P are mutually tangent at P.

2. Let ABCD be a convex quadrilateral with no-parallel sides BC and AD. Assume that there is a point E on the side BC such that the quadrilaterals ABED and AECD are circumscribed. Prove that there is a point F on the side AD such that the quadrilaterals ABCF and BCDF are circumscribed if and only if AB is parallel to CD.

3. Let ABC be triangle with circumcenter O and I. The points D, E and F on the sides BC, CA, and AB respectively are such that $BD + BF = CA$ and $CD + CE = AB$. The circumcircles of the triangles BFD and CDE intersect at $P \neq D$. Prove that $OP = OI$.

4. Let ABC be triangle with $\angle BCA = 90°$, and let C_0 be the foot of the altitude from C. Choose a point X in the interior of the segment $CC_0$and let K, L be the points on the segments AX, BX for which $BK = BC$ and $AL = AC$ respectively. Denote by M the intersection of AL and BK. Show that $MK = ML$.

5. ABCDEF is a convex hexagon, such that in it $AC \parallel DF$, $BD \parallel AE$ and $CE \parallel BF$. Prove that $AB^2 + CD^2 + EF^2 = BC^2 + DE^2 + AF^2$.

6. The point A_1on the perimeter of a convex quadrilateral ABCD is such that the line AA_1 divides the quadrilateral into two parts of equal area. The points B_1, C_1, D_1 are defined similarly. Prove that the area of the quadrilateral $A_1B_1C_1D_1$ is greater than a quarter of the area of ABCD.

7. Prove that if a convex pentagon satisfies the following conditions, then it is a regular pentagon :

   (a) all the interior angles of the pentagon are congruent;

   (b) the lengths of the sides of the pentagon are rational numbers.

8. In a triangle ABC, the bisector of angle BAC meet BC at D. Suppose that $BD \cdot CD = AD^2$ and $\angle ADB = \frac{\pi}{4}$. Determine the angles of triangle ABC.

9. The points A_1, B_1, C_1 are the midpoints of sides BC, CA, AB of acute triangle ABC. On lines B_1C_1 and A_1B_1 are chosen points E and F such that line BE bisects angle AEB_1 and line BF bisects angle CFB_1. Prove that $\angle BAE = \angle BCF$.

10. Acute triangle ABC has incenter I and orthocenter H. The point M is the midpoint of minor arc AC of the circumcircle of ABC. Given that $MI = MH$, find $\angle ABC$.