GEOMETRI

Summer Olympiad Camp 2017 - Matematika SMP

1. In the triangle ABC, $\angle C = 90°$, $2AC > AB$, points E and F on AC and AB respectively are such that $CE = EF$. Suppose EF is the segment of minimal length that divides the area of triangle ABC into two equal halves. Given $AC = 6 + 3\sqrt{3}$, find the length of AB.

2. Let ABCD be a square . Let M an inner point on side BC and N be an inner point on side CD with $\angle MAN = 45°$. Prove that the circumcentre of AMN lies on AC.

3. Three circular arcs w_1, w_2, w_3 with common endpoints A and B are on the same side of line AB; w_2 lies between w_1 and w_3. Two rays emanating from B intersect these arcs at M_1, M_2, M_3 and K_1, K_2, K_3 respectively. Prove that $\frac{M_1M_2}{M_2M_3} = \frac{K_1K_2}{K_2K_3}$.

4. Given a parallelogram ABCD. A circle passing through A meets the line segments AB, AC and AD at inner points M, K, N respectively. Prove that $|AB| \cdot |AM| + |AD| \cdot |AN| = |AK| \cdot |AC|$.

5. The points A, B, C, D, E lie on the circle c in this order and satisfy $AB \parallel EC$ and $AC \parallel ED$. The line tangent to the circle c at E meets the line AB at P. The lines BD and EC meet at Q . Prove that $|AC| = |PQ|$.

6. D is the midpoint of the side BC of the given triangle ABC. M is a point on the side BC such that $\angle BAM = \angle DAC$. L is the second intersection point of the circumcircle of the triangle CAM with the side AB. K is the second intersection point of the circumcircle of the triangle BAM with the side AC. Prove that $KL \parallel BC$.

7. The edges of regular hexagon ABCDEF are made of mirrors. A laser fired from toward the interior of edge $\overline{CD}$, striking it at at point G. The laser bema reflects off the interior of exactly one additional edge and returns to A. Compute $\tan(\angle DAG)$.

8. In the acute-angled triangle ABC, the foot of the perpendicular from B to CA is E. Let l be tangent to the circle ABC at B. The foot of the perpendicular from C to l is F. Prove that EF is parallel to AB.

9. Let ABC be an equilateral triangle, and P be a point inside this triangle. Let D, E and F be the feet of the perpendiculars from P to the sides BC, CA and AB respectively. Prove that

   (a) $AF + BD + CE = AE + BF + CD$ and

   (b) $[APF] + [BPD] + [CPE] = [APE] + [BPF] + [CPD]$.

   The area of triangle XYZ is denoted $[XYZ]$

10. Two circles S and T touch at X. They have a common tangent which meets S at A and T at B. The points A and B are different. Let AP be a diameter of S. Prove that B, X and P lie on a straight line.